## Behavioral Economics

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## Semester Plan

Course Overview
I. What is Behavioural Economics
II. Principles of Experimental Economics
III. The Standard Economic Model: Consumer Theory
IV. Reference dependence \& departures from the standard model
V. Decisions Under Risk and Uncertainty
VI. Intertemporal Choice
VII. Interaction with others: Game Theory
VIII.Interaction with others: Social Preferences

## Today

V. Decisions Under Risk and Uncertainty

Preliminaries (notation and how to set up the problem)

- Expected Value Theory and the St. Petersburg Paradox
- Expected Utility Theory: axioms and representation theorem
- Risk aversion and risk premium
- Application: Jen's stock market decision
- Limitations of Expected Utility Theory


## Decisions under certainty

Given a budget and prices what combination of goods makes a consumer the happiest? Everything is known and there is no uncertainty about how actions lead to outcomes


[^0]
## But.. Uncertainty pervades our decisions



Should I buy a brand new phone (safe but expensive) or a second hand one (risky but saves money)?


Should I buy an extended warranty or will the phone prove robust?


Should I schedule a hike trip in the weekend or will it rain?


Our decisions are almost always involving some degree of uncertainty

In this lecture we introduce some tools that will help us tame it

## A simple case study



- Every day he has to decide which product to sell the next day.
- His earnings depend on the next day's weather. In a rainy day, he earns more if he sells umbrellas. In a sunny day he earns more if he sells hats.
- The decision has to be made the previous day, so there is uncertainty regarding the weather.
- What should the vendor do?


## Step 1: Express the problem in matrix form

Net profits obtained from merchandise, depending on weather

|  | Sunny | Rain |
| :--- | :--- | :--- |
| Sell Umbrellas | $\$ 36$ | $\$ 81$ |
| Sell Hats | $\$ 144$ | $\$ 0$ |

## Street vendors payoff table

- Net profits obtained from merchandise, depending on weather

|  | Sunny | Rain |
| :--- | :--- | :--- |
| Sell Umbrellas | $\$ 36$ | $\$ 81$ |
| Sell Hats | $\$ 144$ | $\$ 0$ |

- Question: What should the vendor do?
- MaxiMin: Choose the action that maximises the worst possible payoff
- Sell always umbrellas: Can be too pessimistic...
- MaxiMax: Choose the action that maximizes the best possible payoff
- Sell always hats: Can be too optimistic
- Question: What information are we missing?


## Street vendors payoff table with probabilities

- Net profits obtained from merchandise, depending on weather

|  | Sunny $\left(\mathrm{p}_{\text {sunny }}=\mathbf{0 . 5}\right)$ | Rain $\left(\mathrm{p}_{\text {rain }}=\mathbf{0 . 5}\right)$ |
| :--- | :--- | :--- |
| Sell Umbrellas | $\$ 36$ | $\$ 81$ |
| Sell Hats | $\$ 144$ | $\$ 0$ |

Decisions under risk: where probabilities and outcomes are well known. It is a special case of decisions under uncertainty.

The matrix representation of the decision is also called: "state contingent representation"

## Prospect notation

- 'Prospects' (often referred to as 'lotteries' or 'gambles'): probability distributions over (monetary) outcomes.
- $L=\left(x_{1}, p_{1} ; x_{2}, p_{2} ; \ldots ; x_{i}, p_{i} ; \ldots ; x_{n}, p_{n}\right)$, where $x_{i}$ is the ith outcome and $p_{i}$ the probability corresponding to the event associated with this outcome. We also impose that $p_{i}>0 \forall i$ and $\Sigma p_{i}=1$.
- For convenience, we order outcomes so that $x_{1}>x_{2}>\cdots>x_{n}$
- Binary prospects of the type: $(x, p ; y, 1-p)$ are often simply notated as: $(x, p ; y)$


## Certainty equivalent

- Certainty Equivalent (CE): The certain amount of money that makes an agent indifferent between the prospect or the certain amount.
- We write $\mathrm{CE}(\mathrm{L})=\$ \mathrm{x}$ and read: the amount of money that makes someone indifferent between keeping or selling the lottery
- You can think of the $\mathrm{CE}(\mathrm{L})$ is equivalent to the minimum price you would be willing to sell lottery: L if you previously owned it (i.e. "willingness to accept").
- You can also think of it as the maximum price you would be willing to pay in order to buy the lottery, if you did not own it before ("willingness to pay").
- In principle, willingness to accept should be equal to your willingness to pay (we will return to this point)


## Step 2: express the problem in prospect notation

- The street vendor chooses between:
- Sell umbrellas: $\mathrm{L}_{U m b}=(\$ 81,0.5 ; \$ 36,0.5)$, or simply $\mathrm{L}_{U m b}=(\$ 81,0.5 ; \$ 36)$
- Sell hats: $\mathrm{L}_{\text {Hat }}=(\$ 144,0.5 ; \$ 0,0.5)$, or simply $\mathrm{L}_{\text {Hat }}=(\$ 144,0.5 ; \$ 0)$
- Notice that in prospect notation, states are no longer represented. Under the standard model, only outcomes matter, not the state in which they are realised.
- How does he decide which option he prefers?
- $L_{U m b}>L_{\text {Hat }}$ or $L_{\text {Hat }}>L_{U m b}$ or $L_{U m b} \sim L_{\text {Hat }}$ ???
- Approach 1: choose the option with the highest Expected Value


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## Expected Value

- The expected value $\mathrm{EV}(\mathrm{L})$ of prospect L is the probability weighted sum of outcomes.
- $E V(L)=\Sigma_{i} p_{i} x_{i}=p_{1} x_{1}+\cdots+p_{n} x_{n}$
- $\$ E V(U m b r)=0.5 * 81+0.5 * 36=40.5+18=58.5$
- $\$ E V(H a t)=0.5 * 144+0.5 * 0=72+0=72$
- $E V(H a t)>E V(U m b r) \Rightarrow L_{H a t}>L_{U m b} \Rightarrow$ sell hats.
- Question: Is this the only legitimate advice? Would the vendor be "wrong" if he chose to sell umbrellas instead?


## Thought experiment

- Question: Which option do you prefer?
- $\quad R=(\$ 1000,0.51 ; \$ 0,0.49)$ or $\mathrm{S}=(\$ 500,1) ?$
- In words: do you prefer option $R$, offering $\$ 1000$ with $51 \%$ chance and $\$ 0$ otherwise, or option $S$, offering $\$ 500$ for sure?
- The EV of $R=0.51 * 1000+0.49 * 0=510>500$. Therefore, according to EV, one "should" choose R. Nonetheless, most people "prefer" the safe (S) option.


## The St. Petersburg paradox

- A fair coin is tossed until Tails appear.
- You receive $2^{n}$ dollars if the first tail occurs on trial $n$
- Question: How much are you willing to pay in order to participate?
- In other words, what is your CE of this lottery?
- $\mathrm{EV}: \frac{1}{2} * 2^{1}+\frac{1}{2^{2}} * 2^{2}+\frac{1}{2^{3}} * 2^{3}+\cdots=1+1+1 \ldots=\infty$
- Yet, most people are not willing to pay more than $\$ 5$ to participate.


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## The birth of a new theory

"The determination of the value of an item must not be based on the price, but rather on the utility it yields.... There is no doubt that a gain of one thousand ducats is more significant to the pauper than to a rich man though both gain the same amount."


Daniel Bernoulli (1700-1782)

## Expected Utility Theory (EUT)

- Bernoulli: Instead of monetary values (x) he proposed to use intrinsic values (utilities, $\boldsymbol{u}(\boldsymbol{x})$ ) of these monetary values.
- Note: $x=$ wealth + outcome of the lottery; for simplicity, we take wealth= 0 from now on.
- Therefore, people valuate prospect, $\mathrm{X}=\left(\mathrm{x}_{1}, \mathrm{p}_{1} ; \mathrm{x}_{2}, \mathrm{p}_{2} ; \ldots x_{i}, p_{i}, \ldots x_{n}, p_{n}\right)$, not as:

$$
\mathrm{EV}(\mathrm{X})=\Sigma_{i} p_{i} x_{i}
$$

- but as

$$
\mathrm{EU}(\mathrm{X})=\Sigma_{i} p_{i} u\left(x_{i}\right)
$$

- Notice: expected value theory is a special case of expected utility theory, where $u(x)=x$


## EUT and diminishing marginal utility

- Diminishing marginal utility: Intrinsic worth of money increases with money, but at a diminishing rate.
- Question: what type of function has this property?
- Concave functions:
- $\frac{d(U(x))}{d x}>0 \& \frac{d^{2}(U(x))}{d x}<0$


## EUT and St. Petersburg Paradox

- Bernoulli suggested a logarithmic utility function: $\mathrm{u}(x)=\ln (x)$
- $U^{\prime}(x)=\frac{1}{x} ; U^{\prime \prime}(x)=-\frac{1}{x^{2}}$, in $(0, \infty)$
- Expected value is replaced by expected utility. So instead of: EV: $\frac{1}{2} * 2^{1}+\frac{1}{2^{2}} * 2^{2}+\frac{1}{2^{3}} * 2^{3}+\cdots=$
- We now write:

$$
\begin{aligned}
E[U(X)] & =\frac{1}{2} U(2)+\frac{1}{4} U\left(2^{2}\right)+\frac{1}{8} U\left(2^{3}\right)+\ldots+\frac{1}{2^{n}} U\left(2^{n}\right)+\ldots \\
& =\frac{1}{2} \ln (2)+\frac{1}{4} \ln \left(2^{2}\right)+\frac{1}{8} \ln \left(2^{3}\right)+\ldots+\frac{1}{2^{n}} \ln \left(2^{n}\right)+\ldots \\
& =\frac{1}{2} \ln (2)+\frac{1}{4}(2 \ln (2))+\frac{1}{2}(3 \ln (2))+\ldots+\frac{1}{2^{n}}(n \ln (2))+\ldots \\
& =\left(\frac{1}{2}+\frac{2}{4}+\frac{3}{2^{3}}+\frac{4}{2^{4}}+\frac{5}{2^{5}}+\ldots\right) \ln 2 \\
& =\left(\sum_{n=1}^{\infty} \frac{n}{2^{n}}\right) \ln 2
\end{aligned}
$$

- Where we used the result that $\ln \left(x^{n}\right)=n \ln (x)$


## EUT and St. Petersburg Paradox (continued)

- $\sum_{n=1}^{\infty} \frac{n}{2^{n}}=2$ (convergent series).
- Therefore the expected utility of this expression is finite: $E U(X)=2 \ln 2=\ln 2^{2}=\ln 4$.
- What about the CE? We know that $u(x)=\ln x$, so we need to solve:
- $u(C E)=\ln 4 \Rightarrow \ln (C E)=\ln 4 \Rightarrow C E=\$ 4$
- This calculation matches empirical data


## Expected Utility Representation

In decisions under certainty:

- Theorem 1: if preferences are rational (i.e. complete and transitive) and continuous then there is a continuous function $u($.$) representing \succcurlyeq$, such that:

$$
\begin{gathered}
x>y \Leftrightarrow u(x)>u(y) \\
x \sim y \Leftrightarrow u(x)=u(y)
\end{gathered}
$$

Furthermore, we can solve problems of the type: $\max (U(x))$ subject to a budget constraint

In decisions under uncertainty we need one additional assumption: Independence (of irrelevant alternatives).

## EUT representation: Independence of Irrelevant Alternatives

- Independence of Irrelevant Alternatives ("Independence" for short) is satisfied if for every $P, Q, R \in L$ and every $\alpha \in(0,1)$

$$
P \succcurlyeq R \Rightarrow \alpha P \not(1-\alpha) Q \geqslant \alpha R+(1-\alpha) Q
$$

- In words, if lottery P is preferred to lottery R then adding to the mix a third lottery, should be irrelevant.
- As we will see, the Independence axiom is the source of most empirical violations of EUT.


## First order stochastic dominance (FOSD)

The matrix representation is also referred to as "state contingent representation".

| Dice <br> Roll | 1 | 2 | 3 | $\mathbf{4}$ | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| B | $\$ 1$ | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| C | $\$ 3$ | $\$ 3$ | $\$ 3$ | $\$ 1$ | $\$ 1$ | $\$ 1$ |

- Lottery A has FOSD over Lottery B as it gives at least as high a probability of receiving at least \$1 as does $B$, and for the outcome $\$ 2$, A gives a higher probability of receiving at least $\$ 2$.
- FOSD is similar to the non-satiation principle from choice under certainty.
- You can think of it as "the more - the better"


## Prospect Notation:

$L_{A}=\left(\$ 1, p_{1}=\frac{1}{6} ; \$ 1, p_{2}=\frac{1}{6} ; \$ 2, p_{3}=\frac{1}{6} \ldots\right)=\left(\$ 2, \frac{2}{3} ; \$ 1, \frac{1}{3}\right)$
$L_{B}=\left(\$ 2, \frac{1}{2} ; \$ 1, \frac{1}{2}\right)$
$L_{C}=\left(\$ 3, \frac{1}{2} ; \$ 1, \frac{1}{2}\right)$

## First order stochastic dominance (FOSD)

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| Dice <br> Roll | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| B | $\$ 1$ | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| C | $\$ 3$ | $\$ 3$ | $\$ 3$ | $\$ 1$ | $\$ 1$ | $\$ 1$ |

## Prospect Notation:

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## First order stochastic dominance (FOSD)

The matrix representation is also referred to as "state contingent representation".

| Dice <br> Roll | $\mathbf{1}$ | 2 | 3 | $\mathbf{4}$ | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| B | $\$ 1$ | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| C | $\$ 3$ | $\$ 3$ | $\$ 3$ | $\$ 1$ | $\$ 1$ | $\$ 1$ |

## Prospect Notation:

$$
L_{A}=\left(\$ 1, p_{1}=\frac{1}{6} ; \$ 1, p_{2}=\frac{1}{6} ; \$ 2, p_{3}=\frac{1}{6} \ldots\right)=\left(\$ 2, \frac{2}{3} ; \$ 1, \frac{1}{3}\right)
$$

- Under EUT, $\succcurlyeq$, do not violate FOSD. That is:
- $L_{A}>L_{B}$ and $L_{C}>L_{B}$ or equivalently
- $E U(A)>E U(B)$ and $E U(C)>E U(B)$, irrespective of the functional form we choose for EU
- Note that the order between A and C, remains a matter of preference under EUT

$$
L_{B}=\left(\$ 2, \frac{1}{2} ; \$ 1, \frac{1}{2}\right)
$$

$L_{B}=\left(\$ 2, \frac{1}{2} ; \$ 1, \frac{1}{2}\right)$

$$
L_{C}=\left(\$ 3, \frac{1}{2} ; \$ 1, \frac{1}{2}\right)
$$

## Statewise dominance

| Result of <br> a dice <br> roll | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lottery A | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| Lottery B | $\$ 1$ | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |

- Statewise dominance is a special case of FOSD. It requires that a lottery dominates another in every possible state of the world.
- Statewise dominance can only be inferred in "state contingent representations"
- Here: Lottery A statewise dominates B because A gives at least as good payoffs in all possible states and gives strictly better outcome in state 3 .


## First order stochastic dominance ("the more - the better")

## Result of

a dice roll

| Lottery B | $\$ 1$ | $\$ 1$ | $\$ 1$ | $\$ 2$ | $\$ 2$ | $\$ 2$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Lottery C | $\$ 3$ | $\$ 3$ | $\$ 3$ | $\$ 1$ | $\$ 1$ | $\$ 1$ |

- BUT: although Lottery $C$ first order stochastically dominates $B$ it does NOT statewise dominate it.
- $\quad$ y yields higher outcomes for states: 1,2,3 but lower for states: 4,5,6.


## Expected Utility Theorem (von Neumann and Morgestern, 1947)

- If $\geqslant$ is complete, transitive, continuous and satisfies the independence axiom then there exists a function $E U()$ such that for every lotteries: $\mathrm{Q}, L$

$$
\begin{gathered}
\mathrm{Q} \succcurlyeq L \Leftrightarrow \mathrm{EU}(\mathrm{Q}) \geq \operatorname{EU}(L), \\
\text { where } \operatorname{EU}(X)=\Sigma_{i} p_{i} u\left(x_{i}\right)
\end{gathered}
$$

- Moreover, in this case, $\operatorname{EU}($.$) is unique up to a positive$ linear transformation. That is, if $\operatorname{EU}(x)$ represents an agent's preferences then so does $g(E U(x))=\alpha+\beta E U(x)$, with $\alpha, \beta \in R$
- A positive linear transformation represents a change of the 0 (performed by $\alpha$ ) and a change of units (performed by $\beta$ ).
- Implication: You can scale u(.) how you like, e.g. 1 and 0 for utilities of "best" and "worst" consequences.


Left: Oscar Morgenstern (1902-1977)
Right: John von Neumann (1903-1954)

## Today

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## Diminishing marginal utility and risk aversion



Consider the prospect: $\mathrm{L}=(x, p ; y)$ What is the Certainty Equivalent of a person with a utility function with diminishing marginal utility?

## Risk aversion in EUT



- Consider the lottery: $(x, p ; y)$
- The EV of this lottery is $p x+(1-p) y$
- The expected utility of a prospect is lower than the utility of its expected value
- Therefore:

$$
C E(x, p ; y)<E V(x, p ; y)
$$

- This is why insurance companies make profits


## Risk premium

- The risk premium is the amount that a risk-averse person would pay to avoid taking a risk.
- For example, an individual may buy insurance to avoid risk.
- Equivalently, the risk premium is the minimum extra compensation (premium) that a decision-maker would require to willingly incur a risk.
- The risk premium is the difference between the expected wealth from the risky stock and the certainty equivalent.


## Risk preferences in EUT



Risk Aversion: CE(L)<EV(L)

- Willing to pay "risk premium" to insure and avoid risk
Risk Neutrality: $\mathrm{CE}(\mathrm{L})=\mathrm{EV}(\mathrm{L})$
- Indifferent between buying insurance or not Risk Seeking: CE(L)>EV(L)
- Will not buy insurance


## Different risk preferences under EUT



## Measuring risk-aversion

The degree of risk aversion is judged by the shape of the utility function over wealth, u(W)


It is positive for risk-averse individuals, zero for risk-neutral individuals, and negative for those who prefer risk. The larger the Arrow-Pratt measure, the more risk averse the individual is.

## Two ‘special' families of utility functions

Constant Absolute Risk Aversion (CARA)

$$
u(x)=-e^{\alpha x}, \text { for } \alpha>0
$$

Then $u^{\prime}(x)=\alpha e^{-\alpha x}$ and $u^{\prime \prime}(x)=-\alpha^{2} e^{-\alpha x}$.
Therefore, the Arrow-Pratt measure is:

$$
\rho(x)=-\frac{-\alpha^{2} e^{-\alpha x}}{\alpha e^{-\alpha x}}=\alpha
$$

Interpretation: the coefficient is not a function of wealth ( x ). So, the individual is holding the same dollar amount in risky assets across all levels of wealth.

## Two ‘special' families of utility functions

Constant relative risk aversion (CRRA):

$$
\begin{gathered}
u(x)=\frac{x^{1-r}}{1-r}, r \geq 0, r \neq 1 \\
u(x)=\ln (x), \text { for } r=1
\end{gathered}
$$

You can verify that the Arrow-Pratt coef. for this family of utility functions is

$$
\rho(x)=\frac{r}{x}
$$

Suggesting that as wealth increases, individuals hold the same proportion of wealth in risky assets

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## Example

- Jen has a concave utility function of $u(W)=\sqrt{W}$
- Her only asset is shares in an Internet start-up company. Tomorrow she will learn her stock's value
- She believes that it is worth $\$ 144$ with probability $2 / 3$ and $\$ 225$ with probability $1 / 3$.
a) What is her expected wealth and her expected utility?
b) Use the Arrow-Pratt measure to determin her risk preferences.
c) How much would she be willing to pay in order to sell the stock today?
d) What risk premium would she pay to avoid bearing this risk?


## a. Expected Wealth (EW) and Expected Utility (EU)

$$
\begin{aligned}
& E W=p \times W_{1}+(1-p) \times W_{2} \\
& =\left(\frac{2}{3} \times 144\right)+\left(\frac{1}{3} \times 225\right) \\
& =96+75=171 \\
& E U=\left[\frac{2}{3} \times U(144)\right]+\left[\frac{1}{3} \times U(225)\right] \\
& =\left[\frac{2}{3} \times \sqrt{144}\right]+\left[\frac{1}{3} \times \sqrt{225}\right] \\
& =\left[\frac{2}{3} \times 12\right]+\left[\frac{1}{3} \times 15\right] \\
& =8+5=13
\end{aligned}
$$

## b. Arrow - Pratt index for Jen

$\rho=-\frac{d^{2} U(W) / d W^{2}}{d U(W) / d W}$
Jen's Arrow-Pratt coefficient is positive, so Jen is risk averse.
From the form, we also see that her utility function is of Constant Relative Risk Aversion
Jen: $U(W)=W^{.5}$
$\frac{d U(W)}{d W}=.5 W^{-.5}$
$\frac{d^{2} U(W)}{d W^{2}}=-.25 W^{-1.5}$
$\Rightarrow \rho_{\mathrm{Jen}}=-\frac{-.25 W^{-1.5}}{.5 W^{-.5}}=\frac{.25 W^{.5}}{.5 W^{1.5}}=\frac{0.5}{W}$

## c. Valuation of her stock

This is equivalent to asking, what is Jen's certainty equivalent. Or, what is the price that she would be willing to sell her stock today (before she finds out whether it is valued at \$144 or \$225) We found out that her EU of holding the stock is:

$$
E U(\text { stock })=13
$$

To find $C E$ (stock), we set $u(C E)=E U($ stock $)$. Since $u(x)=\sqrt{x}$

$$
\sqrt{C E}=13 \Rightarrow C E=\$ 169
$$

## d) Jen's risk premium (P)

We know that Jen is risk averse
Therefore, we expect that Jen would be willing to pay a "risk-premium" in order to avoid uncertainty
The risk premium is the difference between the expected value of an asset and Jen's subjective valuation of it (her certainty equivalent).
Therefore:

$$
P=E W-C E=171-169=\$ 2
$$

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Limitations of Expected Utility Theory

## Allais' paradox (common consequence)

| Prospect A |
| :---: |
| $\$ 2,500$ with probability 0.33, |
| $\$ 2,400$ with probability 0.66, |
| $\$ 0$ with probability 0.01 |




Most people choose B over A but C over D. This violates Independence.

## Allais' paradox (common consequence)



## Prospect D

$\$ 2,400$ with probability 0.34 , $\$ 2,400$ with probability 0.66 $\$ 0$ with probability 0.

Shift 0.66 probability from the 0 outcome in $C \& D$ towards a $\$ 2,400$ outcome makes $C \& D$ equivalent $A \& B$. According to "Independence", the preference should had been maintained.

## Allais' paradox (common consequence)

$$
\begin{align*}
& \text { Preferring Prospect B over Prospect A implies that } E U(B)>E U(A) \Rightarrow \\
& \qquad u(2400)>0.33 u(2500)+0.66 u(2400)+0.01 u(0) \Rightarrow \\
& 0.34 u(2400)>0.33 u(2500) \\
& \text { Preferring Prospect C over Prospect D implies that } E U(C)>E U(C) \Rightarrow \\
& 0.33 u(2500)>0.34 u(2400)
\end{align*}
$$

Clearly, I and II cannot be true at the same time.

## Preference for lotteries and insurance



## Prospect G

\$-5,000 with probability 0.001 , $\$ 0$ with probability 0.999

- Most people choose: E over F, but H over G.

- Similarly, empirical evidence suggest that people play in national lotteries but also buy insurance.
- Risk seeking for small probability of big gains but risk aversion for small probability of big losses.
- Problem as according to EUT, preferences should be stable...


## Endowment effect

- Consider your preferences for coffee mugs.
- Experiments show that WTA(mug)>WTP(mug).
- That is, the minimum amount of money one is willing to accept in order to sell a mug he owns, is higher than the amount of money he is willing to give in order to buy a mug he doesn't own.
- Why is this a problem for the standard model?
- Notice, this scenario involves riskless choices.

$\operatorname{Median}\left(\frac{\text { Mean } W T A}{\text { Mean } W T P}\right)=2.6$


## Endowment effect

- Suppose your utility function over mugs is given by

$$
u(x)=3 \sqrt{x}
$$

- The amount of utility of receiving one mug from 0 mugs is:

$$
u(1)=3
$$

- Which implies that you would be willing to pay a $\$$-amount equivalent to 3 utils to receive your first mug (WTP).
- Similarly, the amount of utility lost from giving away that first mug is:

$$
u(0)-u(1)=0-3=-3
$$

- Which implies that you would be willing to accept a \$compensation equivalent to 3 utils to give this mug away (WTA).
- Therefore, according to the standard model WTP=WTA
- But: WTA>WTP.



[^0]:    Prof. Dr. Sebastian Goerg \& Dr. Orestis Kopsacheilis | Behavioral Economics

