Behavioral Economics

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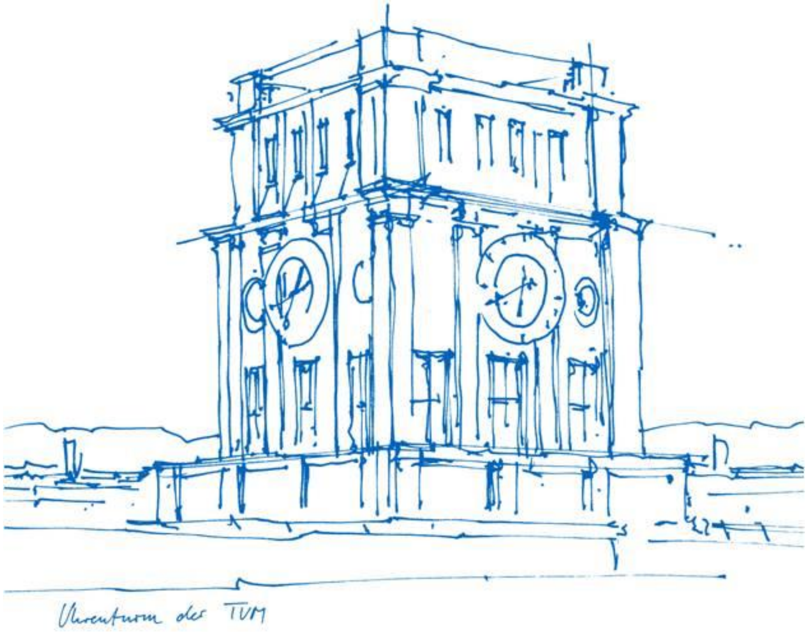

Semester Plan

Course Overview

Emphasis on individual decisions

I. What is Behavioural Economics

II. Principles of Experimental Economics

III. The Standard Economic Model: Consumer Theory

IV. Reference dependence & departures from the standard model

V. Decisions Under Risk and Uncertainty

VI. Intertemporal Choice

VII. Interaction with others: Game Theory

VIII. Interaction with others: Beh. Game Theory & Social Pref/ces

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Emphasis on interactions with others

VII. Interaction with others: Game Theory

VIII. Interaction with others: Beh. Game Theory & Social Pref/ces

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- VII. Interaction with others: Game Theory
- VIII. Interaction with others: Beh. Game Theory & Social Pref/ces

Today

VII. Interactions with others: Game Theory

- Introduction and historical background
- Characterising a game
- One-shot, simultaneous games
  - The Prisoner’s Dilemma and Nash equilibrium
  - Coordination games
  - Mixed Strategies
- Dynamic and Repeated games
  - Backwards induction
  - Market entry game
  - Sub-game perfect Nash equilibrium

What is a game

- We know many different kinds of games: card games, board games, video and computer games, sport games, etc.

- **Game Theory** *does not* focus on these “casual” games, but on general strategic and interactive situations:
    - between two or more participants,
    - each affects the others with her actions,
    - with different possible plays and outcomes,
    - where strategic planning is important.

Applications of Game Theory

Economics

- competition between firms and markets,
- R&D investments,
- auctions,
- trading agreements,
- common pool resources, etc...

Other disciplines

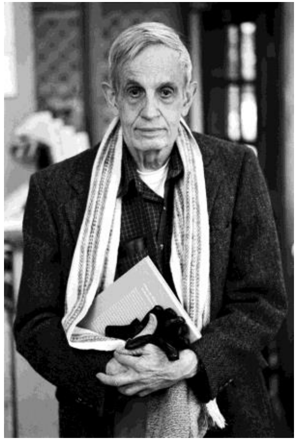
- in biology (dynamics of animal populations),
- in sociology and social psychology (dynamics of groups of people),
- in anthropology (functioning of primitive human societies),
- in political sciences (elections),
- in military sciences (strategic armament),
- in communication sciences (strategic marketing),
- in informatics (networks of autonomous machines),

Historical background

- 1838: Antoine Augustin Cournot considered a duopoly (2-firm market). His analysis concludes with an outcome similar to what is later known as Nash equilibrium.

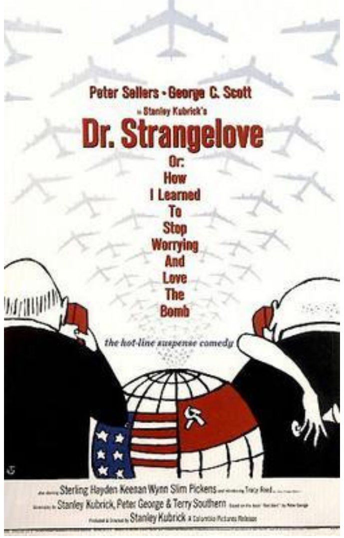
John von Neumann (1903-1957): 2-person zero-sum games analysis (1928 & 1944). These are games where only 3 outcomes are possible: Win, Draw or Loss.

Historical Background



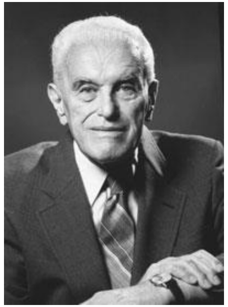
John Forbes Nash (1928-2015; Nobel: 1994) Generalises the notion of equilibrium beyond zero-sum games and for more than 2 players (1950).

- 1950’s: heyday of game theory research in RAND institute – in hopes that the theory would grant US an advantage over USSR (cold-war).



Historical Background

Renhard Selten (1930-2016; Nobel: 1994) subgame perfect Nash equilibria for dynamic games



John Harsanyi(1920-2000; Nobel: 1994) games with asymmetric information

Lloyd Shapley (1923-2016; Nobel: 2012) : cooperative game theory

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Elements of a game

The players: i,j,…n

The rules:

- Order of players decisions
- Feasible decisions at each decision point
- Information at each decision point

Preferences

- Players have preferences over outcomes in the game
- Preferences are modelled just as in decision theory, through a utility function that assigns real numbers to actions.

Taxonomy of games

Cooperative game theory

Assumes:

- rational strategic behavior,
- Unlimited communication,
- unlimited ability to make agreements
- Equilibria are pareto-efficient

Noncooperative game theory

Assumes:

- Rational strategic behavior
- Methodological individualism

Terminology

- **Mutual knowledge**: something that all players know
- **Common knowledge**: something that everyone knows and everyone know that everyone knows, etc…
- **Strategy**: an algorithm that tells the player what decision to make at every decision point
- **Best response**: the decision that secures the highest outcome to a player given what other players might have done in a scenario
- (Strictly) **dominant strategy**: a strategy that yields a player (strictly) higher payoff, no matter which decision(s) the other player(s) choose. Rational players must always choose a strictly dominant strategy.
- **Nash equilibrium**: an outcome where every player is best responding to others

Common knowledge

this entire episode was comedy at its finest

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The prisoner’s dilemma

- Two prisoners are held in separate cells, serving one year for a minor crime. The police, however, suspects them guilty of a larger crime, but have no proof. Each is simultaneously given an opportunity to implicate the other prisoner in the larger crime, in which case his own sentence is reduced by one year and the sentence of the other prisoner increased by three years.
- Players: Prisoner 1, Prisoner 2
- Decisions: cooperate (do not implicate), defect (implicate)
- Preferences: Each prisoner wants to stay as little time as possible in prison. So:
  - $u(4\ years) < u(3\ years) < u(1\ years) < u(0\ years)$
  - This game can be re represented in strategic or matrix form:

Strategic form (matrix representation)

- Prisoner 1 is the row player while Prisoner 2 is the column player.
- We typically write the payoff of the row player in the first cell and that of the column pl. in the second.
- For example, in the combination of strategies: (Cooperate, Defect), the row player receives utility -4 while the column player receives utility 0.
- Can we predict what the two prisoners will do?
- The notion of ***Nash equilibrium*** gives us a prediction.

		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Cooperate	-1,-1	-4,0
	Defect	0,-4	-3,-3

Nash Equilibrium (NE)

Definition: An outcome (combination of strategies) is a ***Nash equilibrium*** of the game when each player takes the action that is her best response to the action taken by her opponents.

- Nash equilibrium: A type of “rational expectations” prediction for the outcome of a game.
- The above is a useful definition because it also provides us with a method of identifying NE.
  - We start by fixing the action of one player (e.g. the column player) and ask: “what is the best response of the other player to her opponent’s action?”

Nash Equilibrium (NE)

- In this example we start by assuming that Prisoner 2 Cooperates.
- Prisoner 1 considers how to best respond. Should she Cooperate, in which case her payoff would be -1 or defect, in which case her payoff is 0? $0 > -1$, therefore she prefers to **Defect**.
- We mark the payoff corresponding to her best action with a circle.

		Prisoner 2
		Cooperate
Prisoner 1	Cooperate	-1,-1
	Defect	0,-4

Nash Equilibrium (NE)

- What if the column player were to Defect instead?
- Then the row player can either Cooperate (-4) or Defect (-3). $-3 > -4$, therefore the row player will Defect too.
- We mark again with a circle

		Prisoner 2
		Defect
Prisoner 1	Cooperate	-4,0
	Defect	-3,-3

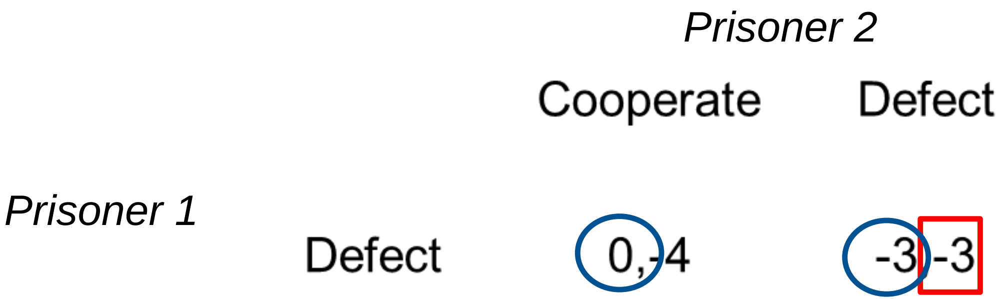
Nash Equilibrium (NE)

- Notice that for the row player “Defect” was the best response no matter whether the column player chose (Defect or Cooperate).
- Therefore, we say that “Defect” is the dominant strategy for Prisoner 1.
- We repeat the analysis for the column player, by fixing the action of the row player.
- If the row player Cooperates, the column player prefers to Defect.
- We mark with a square the utility corresponding to this combination of strategies.

		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Cooperate	-1,-1	-4,$\boxed{0}$

Nash Equilibrium (NE)

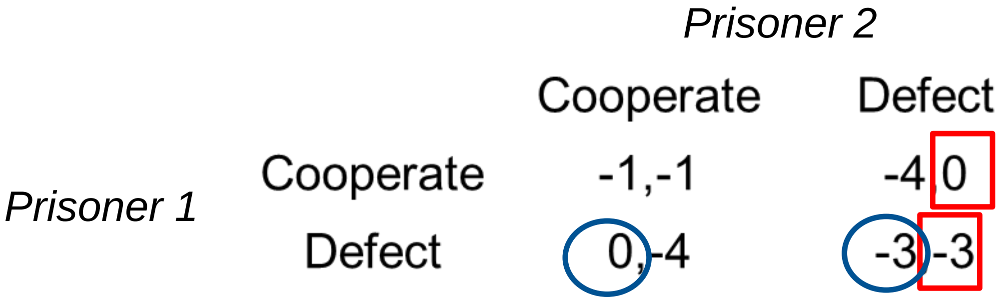
- If the row player Defects, the column player prefers to Defect.
- We mark with a square the utility corresponding to this combination of strategies.
- Again, we find that to Defect is the dominant strategy for the column player



		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Defect	0,-4	-3,-3

Nash Equilibrium (NE)

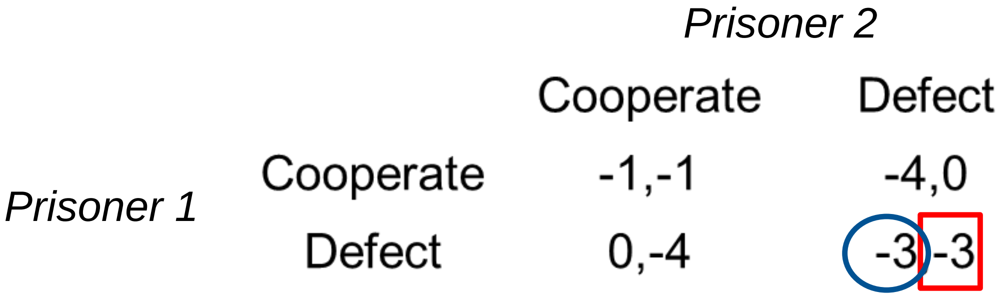
- Putting everything together, the outcome where best responses meet is the Nash equilibrium (according to the definition)
- In this case, NE=(Defect, Defect)
- This means that both prisoners choose to implicate one another and both end up in prison for 3 years (utility corresponding to -3).
- Notice, if they had managed to both stay silent, they would only spend 1 year in prison…



		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Cooperate	-1,-1	-4,0
	Defect	0,-4	-3,-3

Nash Equilibrium (NE)

- Notice, if they had managed to both stay silent, they would only each spend a year in prison.
- In fact, (Cooperate, Cooperate) is strictly ***Pareto dominating*** (Defect, Defect) as it is improving both players' payoff.
- But, mutual cooperation is unachievable. Given Cooperation from one player, the other would always prefer to defect and walk away without any prison time. Therefore, mutual cooperation is ***not*** a NE given our assumptions.



		Prisoner 2	
		Cooperate	Defect
Prisoner 1	Cooperate	-1,-1	-4,0
	Defect	0,-4	-3 -3

Prisoner’s dilemma with different payoff matrix

- Notice that utility is ordinal and therefore, outcomes matter only in their preference ordering.
- Let R = payoff of a player when both cooperate, T=payoff when defecting to a cooperative player, S=payoff when cooperating to a player that defects and P= payoff when the other player also defected.
- Any variation of the matrix that satisfies $T > R > P > S$ satisfies the criteria for a prisoner’s dilemma
- For example:

	Cooperate	Defect
Cooperate	3,3	0,5
Defect	5,0	1,1

Tragedy of the commons

- Tragedy of the commons: The prediction that both players end up implicating one another although a better outcome for both is available if cooperation was possible.
- The game is presented as a story of two prisoner’s but has application well beyond this narrow example. Other examples:
  - Nuclear arms race: arm (D) or disarm (C)?
  - CO2 emissions: keep polluting (D) or go green (C)?
  - Tax evasion: Tax evade (D) or Comply (C)
  - Advertising: Spend on ads (D) or R&D (C)?
  - Sports doping: Take illegal Performance Enhancing Drugs (D) or stay clean (C)?
  - …

- **Social dilemmas**: situations where the strategy that is individually rational is not socially optimal.

A pure coordination game

- Not every game has a unique, dominant strategy, Nash Equilibrium.
- Example: You and your study partner are planning to meet at noon at one of two coffee shops, Lucy’s Coffee and Crestwood Coffee. Unfortunately, you failed to specify which one, and you have no way of getting in touch with each other before noon. If you manage to meet, you get a utility of 1; otherwise, you get a utility of 0.
- Draw the payoff matrix and find the Nash equilibrium or equilibria.

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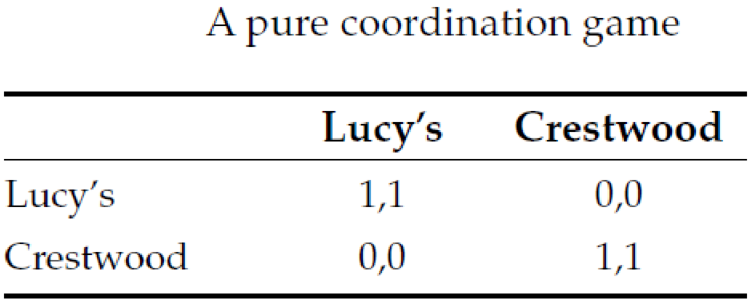

A pure coordination game

	Lucy’s	**Crestwood**
Lucy’s	1,1	0,0
Crestwood	0,0	1,1

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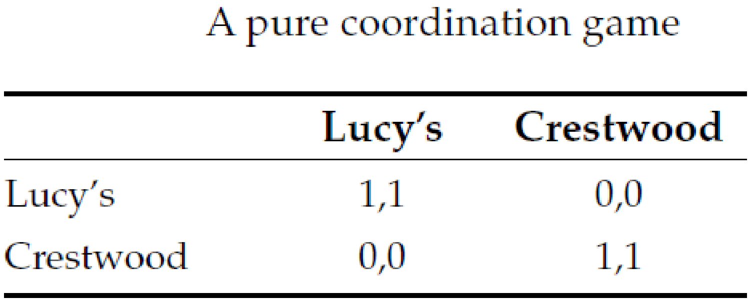

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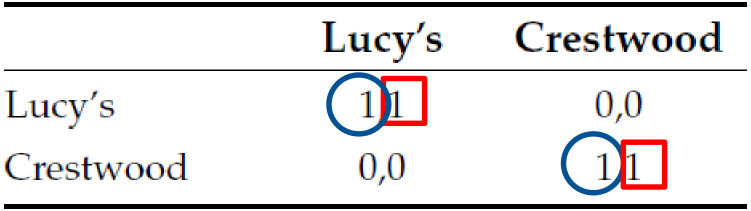
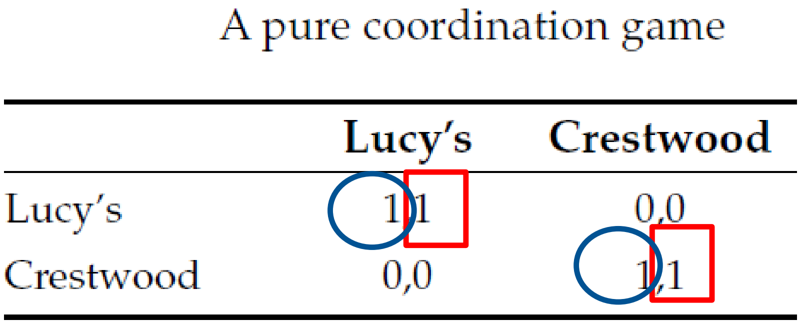


	Lucy’s	**Crestwood**
Lucy’s	1,1	0,0
Crestwood	0,0	1,1

A pure coordination game

- The coffee-shop game is an example of a pure coordination game: a game in which the players’ interests are perfectly aligned.
- In absence of any prior communication, the probability of coordination is 50%



A pure coordination game

	Lucy’s	**Crestwood**
Lucy’s	1,1	0,0
Crestwood	0,0	1,1

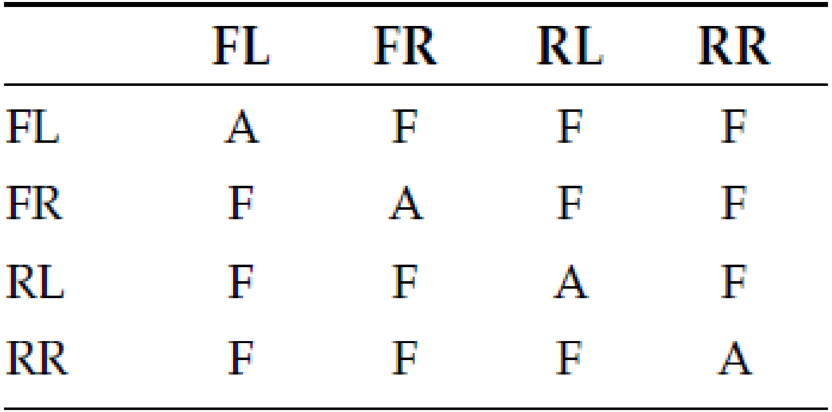
The retake exam – coordination problem

- Bob and Sam are two undergraduate students. The night prior to an important exam they went out clubbing and enjoy themselves a lot (perhaps too much!).
- As a result, they oversleep and did not make it back to campus in time for the exam. So, they called their professor to say that they had got a flat tire on the way to the exam and did not have a spare.
- The professor thought about it and offered them an opportunity to have a retake exam the next day.
- The exam comprised of two parts. Part 1 was fairly easy and was worth 20 points. Bob and Sam were delighted to have scored 20 points easily! Part 2 was comprised of a single question:
- “***Which tire?***” (80 points)

The retake exam – coordination problem

- Bob and Sam’s problem can be modelled in the following matrix.
- FL, FR, RL and RR stand for “Front Left”, “Front Right”, “Rear Left” and “Rear Right” respectively
- “A” is their grade if they manage to coordinate while “F” if not, in which case it is proven that they cheated.

Sam



	FL	FR	RL	RR
FL	A	F	F	F
FR	F	A	F	F
RL	F	F	A	F
RR	F	F	F	A

Bob

Bach or Stravinsky?

- Not every coordination game has perfectly aligned incentives however
- Imagine that two friends, Bob and Sam, agreed to meet this evening, but cannot recall if they will be attending a Bach concert or a Stravinsky concert (and they forgot their phones at home!).
- Bob (row player) would prefer to go to the Stravinsky concert.
- Sam (column player) would rather go to the Bach concert.
- Both would prefer to go to the same place rather than different ones.
- Since they cannot communicate, where should they go?
- The game can be represented in a matrix form:

		Sam	
		Stravinsky	Bach
Bob	Stravinsky	3,2	0,0
	Bach	0,0	2,3

Bach or Stravinsky?

- Our analysis reveals two Nash Equilibria. Both going to Stravinsky or both going to Bach.
- Unlike the pure coordination examples, both outcomes are unfair to one or the other side.
- Assume that Bob and Sam meet in the Stravinsky concert.
- Although Sam would prefer it if *both* switched to Bach, he cannot improve his payoff by *unilaterally* deviating. If he plays Bach when Sam is in Stravinsky, he will end up with a payoff of 0 rather than 2.
- The inability to improve by unilateral deviations is another useful way to think of NE.

		Sam	
		Stravinsky	Bach
Bob	Stravinsky	3,2	0,0
	Bach	0,0	2,3

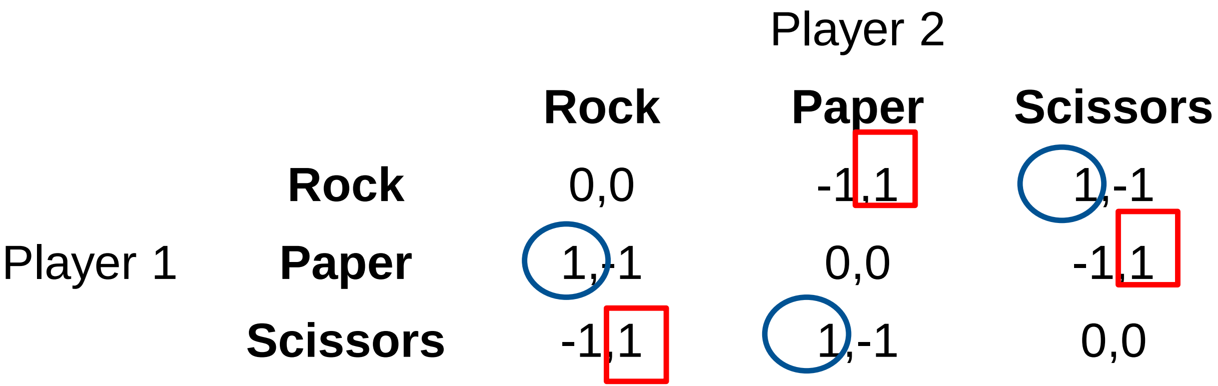
Mixed strategies

- So far, we have seen equilibria in “pure” strategies where moves are played deterministically.
- However, a NE is not guaranteed to always exist in pure strategies.
- Consider the classic: “Rock – papers – scissors” game, where Rock beats Scissors, Scissors beats Paper and Paper beats Rock.
- We can represent it in matrix form:

		Player 2		
		Rock	**Paper**	**Scissors**
Player 1	**Rock**	0,0	-1,1	1,-1
	Paper	1,-1	0,0	-1,1
	Scissors	-1,1	1,-1	0,0

Mixed strategies

- There is no NE in pure strategies
- But there is a NE in mixed strategies, where every player randomises between the three actions with probability: 1/3
  - The formal demonstration is omitted but think intuitively: what would happen if you knew that your opponent was playing Rock more frequently than Paper or Scissors?
- Mixed strategies ensure that “well-behaved” games always have rational-expectations strategy combinations: i.e. that Nash equilibria always exist.



		Player 2		
		Rock	**Paper**	**Scissors**
	Rock	0,0	-1,1	1,-1
Player 1	**Paper**	1,-1	0,0	-1,1
	Scissors	-1,1	1,-1	0,0

Mixed strategies in sports

- Consider the problem the goal-keeper and the kicker face before a penalty kick.

- They both have to choose one of three sides: left, middle or right…
- Football players have been shown to randomise according to game theoretic predictions (Chiappori, Levitt and Gresclose, 2000)
- Similar application: service in tennis

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Dynamic games

So far we looked at **static games**: played once and actions are made simultaneously

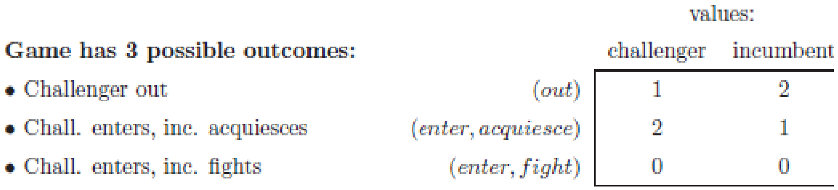
Dynamic games: have several sequential steps and at each step one or more players move

Perfect information and perfect recall: at every step precisely one of the players takes an action, knowing all previous taken actions

(we ignore imperfect information and information asymmetries today)

Market entry: extensive form

Let’s start with a sequential game and two different ways of describing the game:
A challenger wants to enter a market currently monopolized by an active incumbent. If the challenger enters, the incumbent must decide if she accepts or fights it by reducing its prices. The challenger profits by entering without fight, but the incumbent does not, however if they fight they both lose. What’s going to happen in the end?



		values:	
Game has 3 possible outcomes:		challenger	incumbent
• Challenger out	(out)	1	2
• Chall. enters, inc. acquiesces	$(enter, acquiesce)$	2	1
• Chall. enters, inc. fights	$(enter, fight)$	0	0

Normal form game:

		Incumbent	
		acquiesce	*fight*
Challenger	*enter*	2,1	0,0
	out	1,2	1,2

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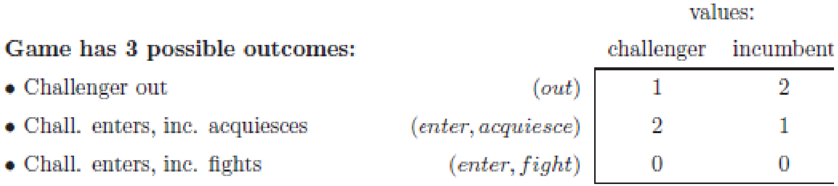

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Normal form game:

		Incumbent	
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Normal form game (strategic/matrix):

- Nash equilibria: (enter, acquiesce) & (out, fight)
- Nash equilibrium (out, fight) is based on a *non-credible* threat.

Incumbent

		acquiesce	*fight*
Challenger	*enter*	$\underline{2}$,$\underline{1}$	0,0
	out	1,2	$\underline{1}$,$\underline{2}$

Market entry: extensive form

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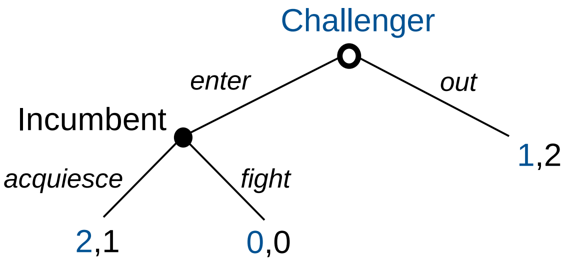
A challenger wants to enter a market currently monopolized by an active incumbent. If the challenger enters, the incumbent must decide if she accepts or fights it by reducing its prices. The challenger profits by entering without fight, but the incumbent does not, however if they fight they both lose. What’s going to happen in the end?

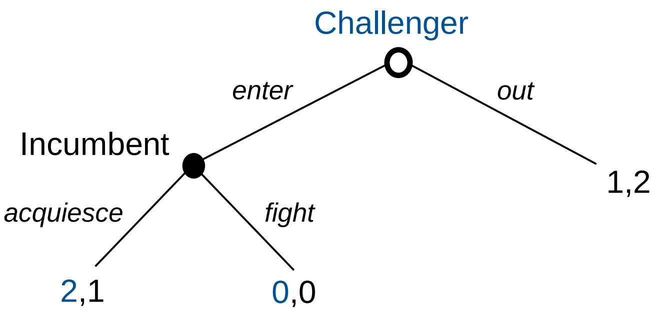
Extensive form game:



Normal form game:

		Incumbent	
		acquiesce	*fight*
Challenger	*enter*	2,1	0,0
	out	1,2	1,2

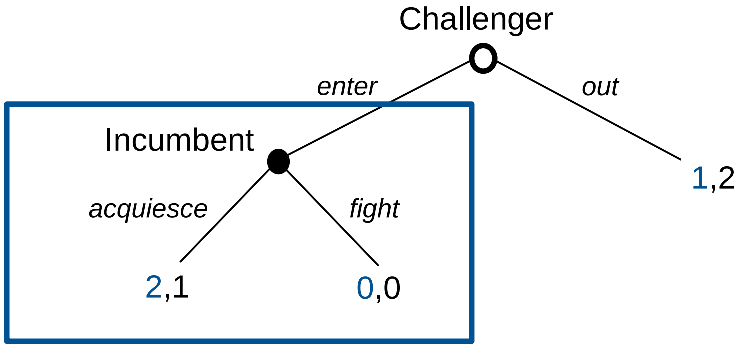
Market entry: extensive form



This game is represented in extensive form:

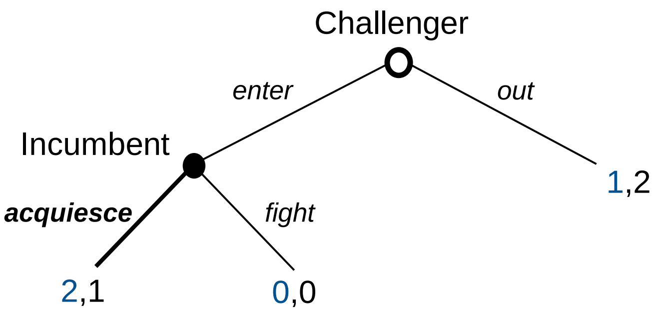
- **nodes**: decision node; each with the name of the player who moves there
- **lines** under a node: all available actions
- **vertical hierarchy**: sequence of decisions
- **numbers**: preferences, the utilities of outcomes

Market entry: extensive form



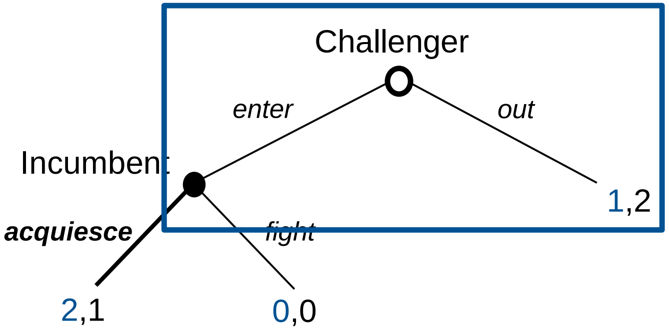
1. Consider the last possible decision (principle of backwards induction).
   - In this decision node, it’s the incumbent who decides to *accept* or to *fight* the challenger.
   - What will the incumbent do?
     - If he choses to accept, his payoff is 1 but if he chooses to fight, then his payoff is 0.
     - 1>0, so the incumbent would choose to accept if the game reaches this decision node.
   - Mark the incumbents decision.

Market entry: extensive form



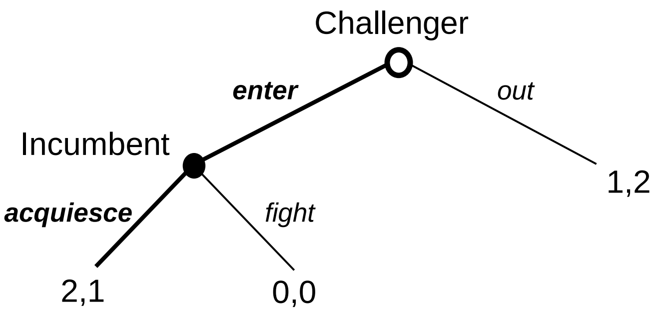
1. Consider the last possible decision (principle of backwards induction).
   - In this decision node, it’s the incumbent who decides to *accept* or to *fight* the challenger.
   - What will the incumbent do?
     - If he choses to accept, his payoff is 1 but if he chooses to fight, then his payoff is 0.
     - 1>0, so the incumbent would choose to accept if the game reaches this decision node.
   - Mark the incumbents decision.

Market entry: extensive form



2. Consider the penultimate decision:
- In this node, it’s the Challenger that decides whether to enter or not (out).
- What will he do?
  - From backwards induction, he know that if he chooses to enter, the incumbent is going to accept him. In this scenario his payoff would be 2. If he chooses not to enter, his payoff would be: 1.
  - 2>1, so the Challenger chooses “enter”.
- Therefore, the outcome is *(enter, acquiesce)*

Market entry: extensive form



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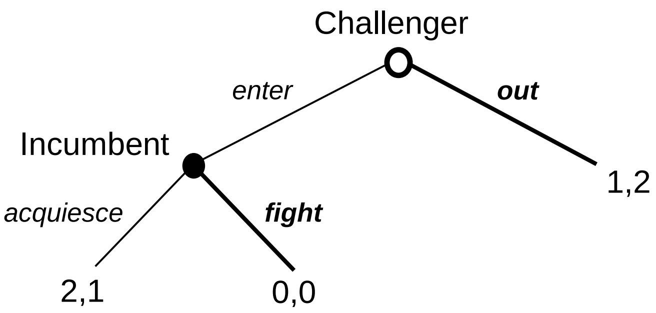
     - 2>1, so the Challenger chooses "enter".
   - Therefore, the outcome is *(enter, acquiesce)*

Market entry: what about the second equilibrium?



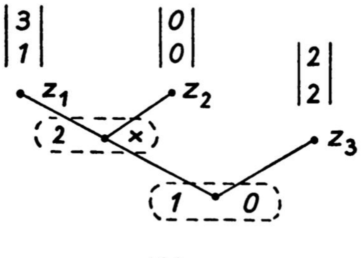
- Nash analysis presumes that players commit to their strategies.

- But in dynamic games this assumption is not fulfilled. The strategy fight is optimal only before the game begins and under expectation that challenger won’t enter. The challenger will realize, however, that the threat of fight isn’t credible: After action enter the incumbent prefers to change her strategy and chose acquiesce.
- A strategy is credible only if it prescribes an optimal action for every decision – *even for one that will not happen*.

Subgame perfect equilibrium

Reinhard Selten (1965) Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit, *Zeitschrift für die gesamte Staatswissenschaft* (*Journal of Institutional and Theoretical Economics*)

Reinhard Selten (1975), Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games, *International Journal of Game Theory*



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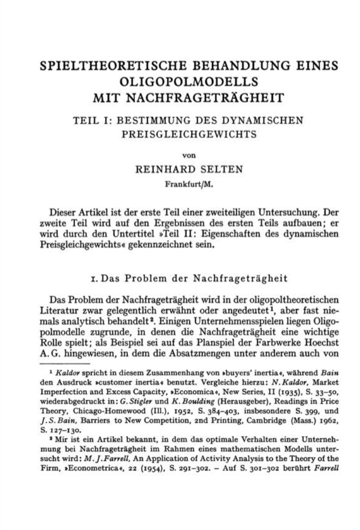

SPIELTHEORETISCHE BEHANDLUNG EINES OLIGOPOLMODELLS MIT NACHFRAGETRÄGHEIT

TEIL I: BESTIMMUNG DES DYNAMISCHEN PREISGLEICHGEWICHTS

von

REINHARD SELTEN

Frankfurt/M.

Dieser Artikel ist der erste Teil einer zweiteiligen Untersuchung. Der zweite Teil wird auf den Ergebnissen des ersten Teils aufbauen; er wird durch den Untertitel »Teil II: Eigenschaften des dynamischen Preisgleichgewichts« gekennzeichnet sein.

1. Das Problem der Nachfrageträgheit

Das Problem der Nachfrageträgheit wird in der oligopoltheoretischen Literatur zwar gelegentlich erwähnt oder angedeutet[1], aber fast niemals analytisch behandelt[2]. Einigen Unternehmensspielen liegen Oligopolmodelle zugrunde, in denen die Nachfrageträgheit eine wichtige Rolle spielt; als Beispiel sei auf das Planspiel der Farbwerke Hoechst A. G. hingewiesen, in dem die Absatzmengen unter anderem auch von

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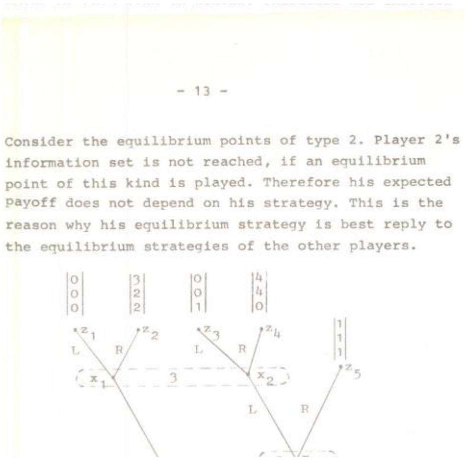
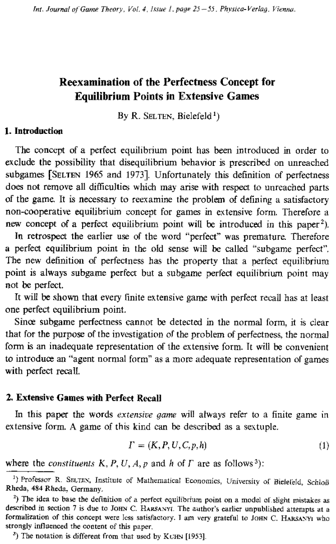


- 13 -

Consider the equilibrium points of type 2. Player 2's information set is not reached, if an equilibrium point of this kind is played. Therefore his expected payoff does not depend on his strategy. This is the reason why his equilibrium strategy is best reply to the equilibrium strategies of the other players.



Int. Journal of Game Theory, Vol. 4, Issue 1, page 25 – 55, Physica-Verlag, Vienna.

Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games

By R. SELTEN, Bielefeld[1])

1. Introduction

The concept of a perfect equilibrium point has been introduced in order to exclude the possibility that disequilibrium behavior is prescribed on unreached subgames [SELTEN 1965 and 1973]. Unfortunately this definition of perfectness does not remove all difficulties which may arise with respect to unreached parts of the game. It is necessary to reexamine the problem of defining a satisfactory non-cooperative equilibrium concept for games in extensive form. Therefore a new concept of a perfect equilibrium point will be introduced in this paper[2]).

In retrospect the earlier use of the word "perfect" was premature. Therefore a perfect equilibrium point in the old sense will be called "subgame perfect". The new definition of perfectness has the property that a perfect equilibrium point is always subgame perfect but a subgame perfect equilibrium point may not be perfect.

It will be shown that every finite extensive game with perfect recall has at least one perfect equilibrium point.

Since subgame perfectness cannot be detected in the normal form, it is clear that for the purpose of the investigation of the problem of perfectness, the normal form is an inadequate representation of the extensive form. It will be convenient to introduce an "agent normal form" as a more adequate representation of games with perfect recall.

2. Extensive Games with Perfect Recall

In this paper the words *extensive game* will always refer to a finite game in extensive form. A game of this kind can be described as a sextuple.

$$\Gamma = (K, P, U, C, p, h) \qquad (1)$$

where the *constituents* K, P, U, A, p and h of Γ are as follows[3]):

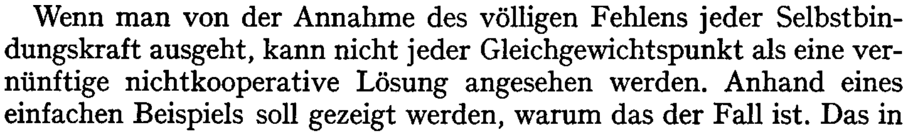
---

[1]) Professor R. SELTEN, Institute of Mathematical Economics, University of Bielefeld, Schloß Rheda, 484 Rheda, Germany.

[2]) The idea to base the definition of a perfect equilibrium point on a model of slight mistakes as described in section 7 is due to JOHN C. HARSANYI. The author's earlier unpublished attempts at a formalization of this concept were less satisfactory. I am very grateful to JOHN C. HARSANYI who strongly influenced the content of this paper.

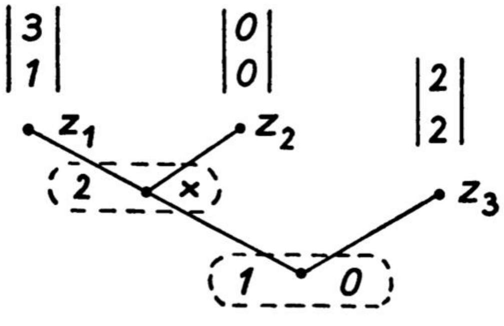
[3]) The notation is different from that used by KUHN [1953].

Subgame perfect equilibrium



Wenn man von der Annahme des völligen Fehlens jeder Selbstbindungskraft ausgeht, kann nicht jeder Gleichgewichtspunkt als eine vernünftige nichtkooperative Lösung angesehen werden. Anhand eines einfachen Beispiels soll gezeigt werden, warum das der Fall ist. Das in

Assuming that a credible commitment is not possible, one cannot consider every equilibrium as a sensible non-cooperative solution. The following simple example should demonstrate this case.



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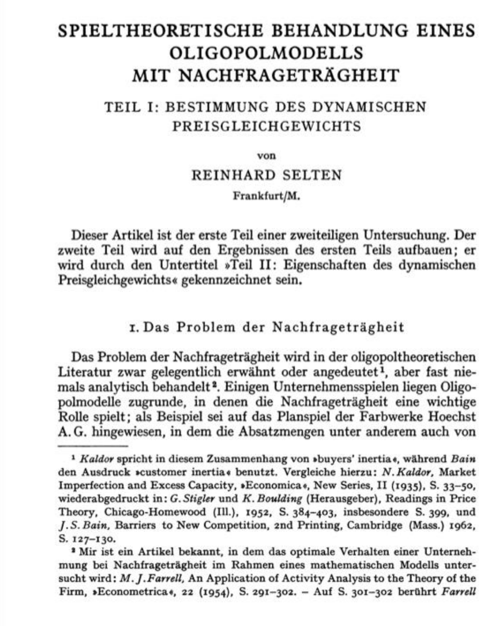

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Subgame perfect equilibrium

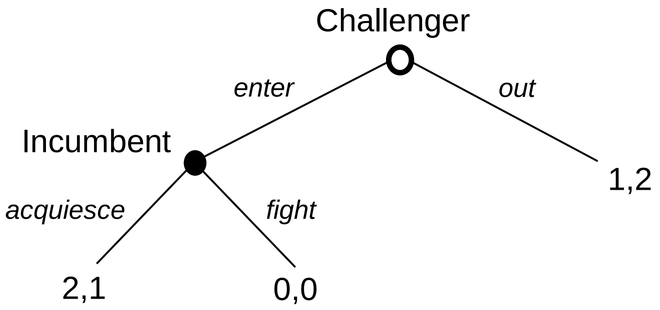
A subgame perfect equilibrium is a **refinement** of a Nash equilibrium. *A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game.*

That means, once I work through the game in chronoligical order and I reach a node, any decision should reflect a Nash equilibrium for the continuation game (subgame).

Looking at the subgames:

A subgame perfect equilibrium is a **refinement** of a Nash equilibrium. *A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game.*

- Every decision node initiates a subgame.
- The game has two decision nodes, therefore it has two subgames:



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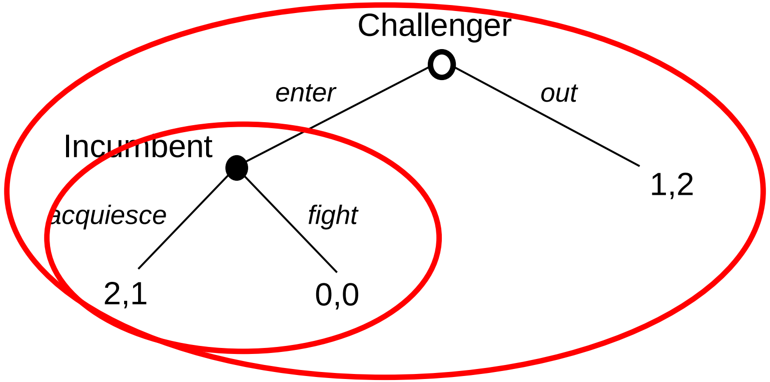

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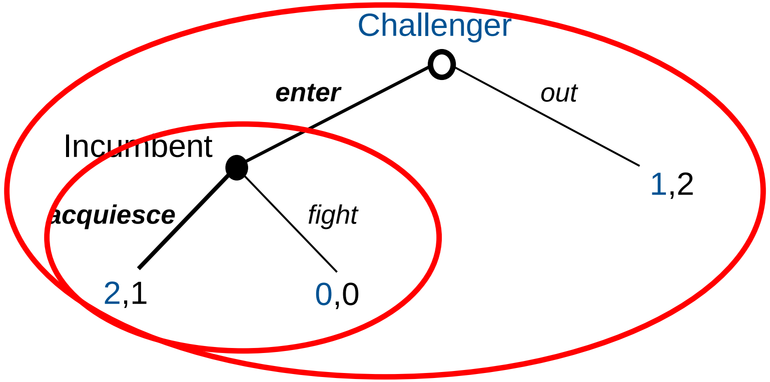
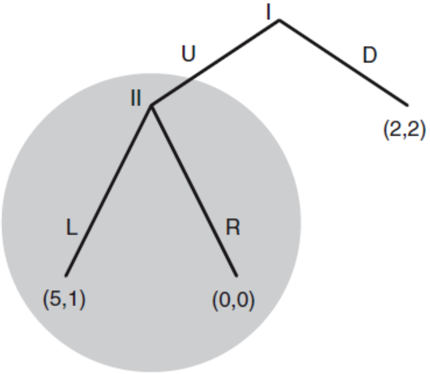


Credible Threat – MAD

(based on Angner (2016) page 236 ff.)

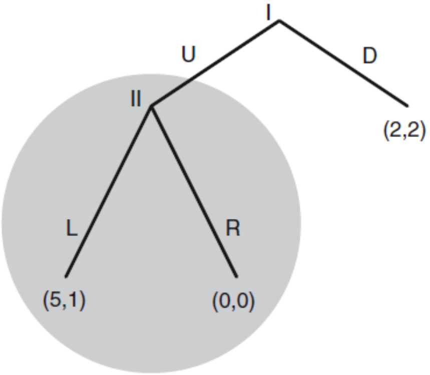
Mutually assured destruction (MAD) is a military doctrine according to which two superpowers (such as the US and the USSR) can maintain peace by threatening to annihilate the human race in the event of an enemy attack. Suppose that the US moves first in the following game:



Credible Threat – MAD

(based on Angner (2016) page 236 ff.)

The US can launch an attack (U) or not launch an attack (D). If it launches an attack, the USSR can refrain from retaliating (L) or annihilate the human race (R). Given the payoff structure of the game in the figure, D, R is a Nash equilibrium. The doctrine is flawed, however, in that the threat is not credible: the MAD Nash equilibrium presupposes that USSR forces are willing to annihilate the human race in the event of a US attack, which would obviously not be in their interest. Thus, the MAD Nash equilibrium is not subgame perfect.



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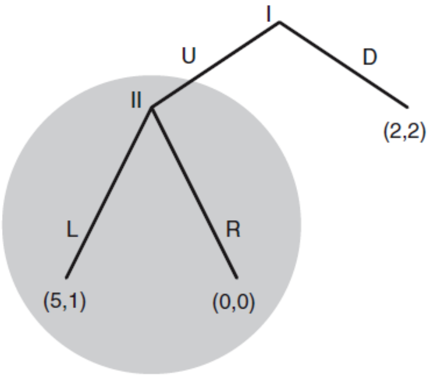

Credible Threat - MAD

In Stanley Kubrick’s 1963 film Dr Strangelove, the USSR tries to circumvent the problem by building a doomsday machine: a machine that in the event of an enemy attack (or when tampered with) automatically launches an attack powerful enough to annihilate the human race. Such a machine would solve the strategic problem, because it guarantees retaliation to enemy attack and therefore makes the threat credible

Peter Sellers as Dr. Strangelove (1969)

Credible Threat - MAD

https://www.youtube.com/watch?v=2yfXgu37iyI

Credible Threat - MAD

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"You see, the whole point of the doomsday machine is lost.... if you keep it a secret! Why didn't you tell the world?!"

Peter Sellers as Dr. Strangelove (1969)

Repeated Games

Repeated games are also a form of dynamic games.

Will you change your behavior because you play the same game repeatedly?

If you play the game a finite number of repetitions (a known number of repetitions that we can count), the Nash equilibrium of a prisoner’s dilemma game remains unchanged: defect/defect

Repeated Games: firm collusion application

Suppose this game is repeated over and over again—for example, you and your competitor simultaneously announce your prices on the first day of every month. Should you then play the game differently?

		Firm 2	
		High Price	**Low Price**
Firm 1	**High Price**	50, 50	-50, 100
	Low Price	100, -50	10, 10

Repeated Games: firm collusion application

Finite number of repetitions

Now suppose the game is repeated a finite number of times, say N months. (N can be large as long as it is finite.) My competitor (Firm 2) is rational and believes that I am rational.

I considering undercutting in last month:

- The other firm cannot retaliate, because the game is over. However, the firm knows that I will charge a low price in the last month. What about the next-to-last month?
- Because there will be no cooperation in the last month, anyway, Firm 2 figures that it should undercut in the next-to-last month.
- But, of course, I have figured this out too. In the end, the only rational outcome is for both of us to charge a low price every month.
- Thus, in a finitely repeated game, the prisoners' dilemma can **not** have a cooperative outcome…Even though both forms **could make much more money** if they found a way to ‚collude'…

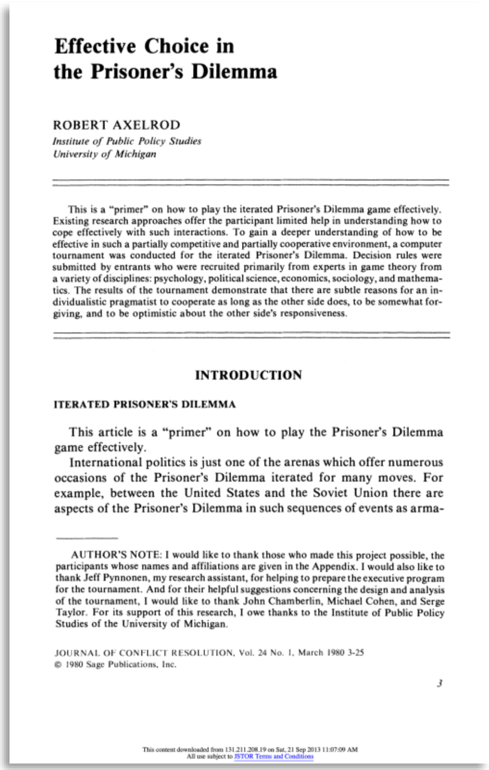
Axelrod Tournaments

1979: Tournament on various strategies for the repeated Prisoner’s Dilemma game by Robert Axelrod (Political Science at the University of Michigan)

- Well-known game theorists to submitted strategies
- Strategies were run by computers, clearly stating when to C or D
- Played for 200 rounds

Examples of submitted strategies:

- **Always defect:** This strategy always defects
- **Always cooperate:** This strategy always cooperates
- **Grim trigger**: Cooperate in the first round and in the subsequent rounds as long as his opponent does not defect from the agreement. If the opponent has defected in the previous round, defect forever.
- **Tit for Tat:** Cooperate in the first round, then do whatever the opponent has done in the previous move
- …. Several other, more complicated strategies!



Effective Choice in the Prisoner’s Dilemma

ROBERT AXELROD
Institute of Public Policy Studies
University of Michigan

This is a “primer” on how to play the iterated Prisoner’s Dilemma game effectively. Existing research approaches offer the participant limited help in understanding how to cope effectively with such interactions. To gain a deeper understanding of how to be effective in such a partially competitive and partially cooperative environment, a computer tournament was conducted for the iterated Prisoner’s Dilemma. Decision rules were submitted by entrants who were recruited primarily from experts in game theory from a variety of disciplines: psychology, political science, economics, sociology, and mathematics. The results of the tournament demonstrate that there are subtle reasons for an individualistic pragmatist to cooperate as long as the other side does, to be somewhat forgiving, and to be optimistic about the other side’s responsiveness.

INTRODUCTION

ITERATED PRISONER’S DILEMMA

This article is a “primer” on how to play the Prisoner’s Dilemma game effectively.

International politics is just one of the arenas which offer numerous occasions of the Prisoner’s Dilemma iterated for many moves. For example, between the United States and the Soviet Union there are aspects of the Prisoner’s Dilemma in such sequences of events as arma-

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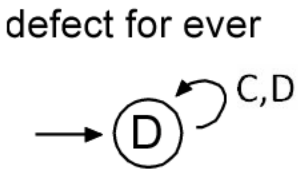
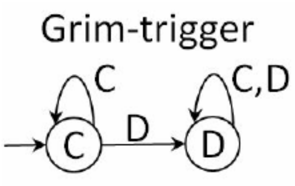
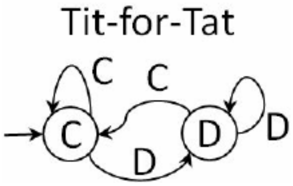
AUTHOR’S NOTE: I would like to thank those who made this project possible, the participants whose names and affiliations are given in the Appendix. I would also like to thank Jeff Pynnonen, my research assistant, for helping to prepare the executive program for the tournament. And for their helpful suggestions concerning the design and analysis of the tournament, I would like to thank John Chamberlin, Michael Cohen, and Serge Taylor. For its support of this research, I owe thanks to the Institute of Public Policy Studies of the University of Michigan.

JOURNAL OF CONFLICT RESOLUTION, Vol. 24 No. 1, March 1980 3-25

Automata







Axelrod Tournaments

Tit-for-Tat by Anatol Rapoport won the competition.

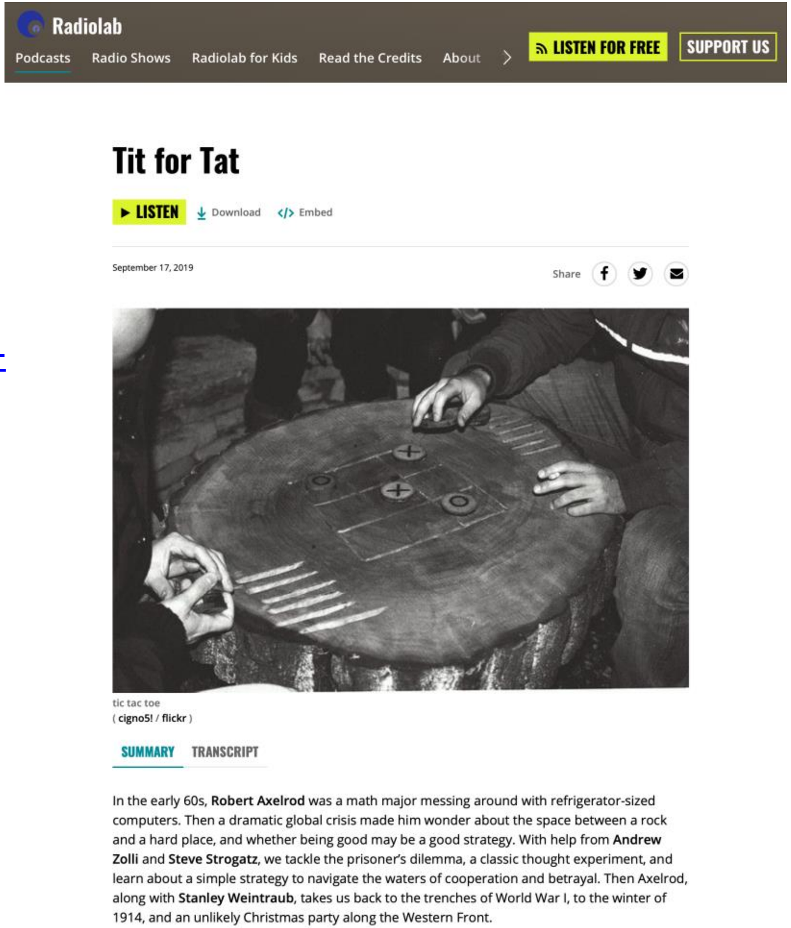
Notable properties of high scoring strategies:

- Nice: don’t defect before the opponent does.
- Retaliating: don’t cooperate blindly or you will be exploited by “nasty” strategies
- Forgiving: eventually fall back to cooperating if the opponent does not continue to defect.
- Non-envious: do not strive to score more than the opponent (pays off in the long run after several encounters)

Important: “best” strategy depends on the environment and the strategies the other players play.

Repeated Prisoner’s dilemma

Very fun and educational podcast episode:
https://radiolab.org/podcast/104010-one-good-deed-deserves-another



Tit for Tat

September 17, 2019

tic tac toe
(cigno5! / flickr)

In the early 60s, **Robert Axelrod** was a math major messing around with refrigerator-sized computers. Then a dramatic global crisis made him wonder about the space between a rock and a hard place, and whether being good may be a good strategy. With help from **Andrew Zolli** and **Steve Strogatz**, we tackle the prisoner’s dilemma, a classic thought experiment, and learn about a simple strategy to navigate the waters of cooperation and betrayal. Then Axelrod, along with **Stanley Weintraub**, takes us back to the trenches of World War I, to the winter of 1914, and an unlikely Christmas party along the Western Front.

Repeated Games: infinite horizon…

Infinitely repeated game

When my competitor and I repeatedly set prices month after month, forever, cooperative behavior (i.e., charging a high price) is then the rational response to a tit-for-tat strategy. (This assumes that my competitor knows, or can figure out, that I am using a tit-for-tat strategy.) It is not rational to undercut.

With infinite repetition of the game, the expected gains from cooperation will outweigh those from undercutting. This will be true even if the probability that I am playing tit-for-tat (and so will continue cooperating) is small.